ISTA Summer School in Dynamical Systems

Program

September 2025

Mini-courses

Geodesics flows on the two-sphere

Alberto Abbondandolo *(Ruhr - Universität Bochum)*

I will discuss some results about the dynamics of geodesic flows on Riemannian two-spheres of revolution : sharp systolic inequalities, Zoll metrics, length spectrum rigidity and flexibility.

On Kolmogorov-typical properties of symplectic dynamics

Pierre Berger *(CNRS - Sorbonne Université)*

We propose a general framework, within which we prove that several properties, such as the fast growth of the number of periodic points, the universality, and the high emergence, hold true for every parameter value for a generic finite-parameter family of symplectic diffeomorphisms displaying an elliptic point. At this occasion we will revisit several classical topics (Birkhov normal form, KAM theorem, renormalization nearby homoclinic point, etc). Joint work with D. Turaev.

KAM normal form

Abed Bounemoura *(CNRS - UPPA)*

We shall introduce basic ideas in KAM theory by showing the existence of quasi-periodic motions within a non-degenerate (Arnold) family of maps of the torus.

Unstable motions in nearly integrable Hamiltonian systems

Marcel Guardia *(Universitat de Barcelona)*

In this course I will explain, through examples, how to construct unstable dynamics in nearly integrable Hamiltonian systems : chaotic motions and Arnold diffusion. A crucial tool will be Melnikov Theory, which analyzes transverse intersections of stable and unstable manifolds of invariant objects (periodic orbits, invariant tori).

On some stochastic properties of Lorentz gases

Dalia Terhesiu *(Universiteit Leiden)*

In these lectures we review old and recent results on stochastic properties of periodic Lorentz gases with finite and infinite horizon, mainly described in terms of the displacement or flight function. For a short definition, one can think of a periodic Lorentz gas as a unit mass particle moving and bouncing elastically in a periodic grid of scatterers. We will focus on some results for the discrete time models (maps) which highlights the diffrence between finite and infinte horizon. In the last part, we also mention recent results for continuous time (flows).

Contributed Talks

Studying network of symmetric periodic orbit families of the Hill problem via symplectic invariants

Cengiz Aydin *(Heidelberg University)*

In this talk I discuss the application of symplectic invariants to analyze the network structure of symmetric periodic orbit families in the framework of the spatial circular Hill three-body problem. The extensive collection of families within this problem constitutes a complex network, fundamentally comprising the so-called basic families of periodic solutions, including the orbits of the satellite g, f, the libration (Lyapunov) a, c, halo and collision $\mathcal{B}_0$ families. The symplectic tools include computation of Conley-Zehnder index, Krein signature, and local Floer homology (graded by Conley-Zehnder indices) and its Euler characteristics. Since the latter is a bifurcation invariant, the computation of Conley-Zehnder indices facilitates the construction of well-organized bifurcation graphs depicting the interconnectedness among families of periodic solutions.

Zoll deformations of Kepler problem

Stefano Baranzini *(Università di Torino)*

In this talk we consider natural Hamiltonians on $\mathbb{R}^2 \setminus \{0\}$ of the form

$$H(p,q) = \frac{1}{2} g^{-1}(p,p)^2 + V(|q|)$$

where $V(q)$ is close to the Kepler potential $-\frac{1}{|q|}$ and g to the standard metric. We build infinite dimensional families of such Hamiltonian enjoying the property that, for any finite number of energies, all negative energy h solution are periodic. Connections with Zoll surfaces of revolution will be discussed.

Multidimensional Birkhoff Theorem for Recurrent Lagrangian Submanifolds

Skander Chafri *(Université Paris Cité)*

A theorem by Birkhoff, established in 1922, states that an essential invariant curve under a twist map of the cylinder is a Lipschitz graph over the circle. Several generalizations of this theorem to higher dimensions have followed. One notable result, due to Marie-Claude Arnaud (2010) concerns

the cotangent bundle of a closed manifold and establishes that any exact Lagrangian submanifold (Hamiltonianly isotopic to the zero section) and invariant under the flow of a Hamiltonian that is convex on the fibers, is a Lipschitz graph over the zero section. More precisely, it is the graph of a weak KAM solution to the Hamilton-Jacobi equation.

We propose an extension of this result to recurrent Lagrangian submanifolds under the action of the Hamiltonian flow, using a topology of convergence that controls the dilations of these submanifolds. In this case, such submanifolds are Lipschitz graphs over the zero section and, more precisely, graphs of differentials of recurrent viscosity solutions.

Periodic Orbits and Contact Type Property in High-Dimensional Magnetic Systems

Lina Deschamps *(Heidelberg University)*

This is joint work with Levin Maier and Tom Stalljohann. We introduce a class of magnetic systems on closed manifolds, called magnetic systems of strong geodesic type, for which there exists at least one null-homologuous embedded periodic orbit on each energy level, of negative energy below the lowest Mañé critical value. This result partially answers a conjecture posed by Contreras–Iturriaga–Paternain–Paternain in the early 2000s.

Continua of nonstationary periodic solutions of Hamiltonian system inspired by cosmology

Agnieszka Gołembiewska *(Nicolaus Copernicus University in Toruń)*

In this talk, we study the existence and properties of continua of nonstationary periodic solutions of Hamiltonian systems. The problem is reformulated in terms of families of 2π-periodic solutions arising in parametrized Hamiltonian systems, where these solutions correspond to critical points of an associated functional. We provide a brief overview of bifurcation theory, addressing both local and global aspects, and emphasize the role of the bifurcation index as a key tool in establishing global bifurcation results. The theoretical framework is illustrated with an application to a Hamiltonian system motivated by problems in cosmology.

Joint typical periodic optimization : expanding maps and beta-transformations

Yinying Huang *(Peking University)*

In the context of ergodic theory of topological dynamical systems, we introduce the notion of joint typical periodic optimization : for a space of maps $\mathcal{T}$, and a function space $\mathcal{F}$, there is an open dense subset of $\mathcal{T} \times \mathcal{F}$ consisting of map-function pairs whose maximizing invariant measure is unique and supported on a periodic orbit. Taking $\mathcal{F}$ to be the Banach space of α-Hölder functions, we establish joint typical periodic optimization theorems for expanding maps on compact locally connected metric spaces, and for beta-transformations on the interval. This is a joint work with Zelai Hao, Oliver Jenkinson. and Zhiqiang Li.

Hausdorff dimension of measures for Interval Exchange Transformations

Łukasz Kotlewski *(Nicolaus Copernicus University in Toruń)*

We study the Hausdorff dimension of the invariant measure for affine interval exchange transformation that is semi-conjugated to a standard self-similar interval exchange transformation. We show that the dimension depends on the decomposition of the log-slopes vector with respect to the eigenvectors of the self-similarity matrix. By exploiting the ergodicity of a Markov chain given by a renormalization operator acting on the associated skew product, we compute the information content of the dynamical partition. This, in turn, leads to explicit formulas for the dimension, which turn out to be closely related to the notions of Kullback–Leibler divergence and unstable entropy. This is joint work with P. Berk, K. Frączek, F. Trujillo. (work in progress)

Abundance of families of diffeomorphisms

Dongchen Li *(Imperial college)*

Let M be a smooth manifold with $\dim(M) \geq 3$ and let V be an open subset of $\mathbb{R}^m$ ($m \geq 1$). Define the space of parametric families of maps $M \to M$:

$$C^{K,r}(V \times M, M) = \{g(t,x) : \partial_t^i \partial_x^j g \text{ exist and are continuos for } 0 \leq |i| \leq K, 0 \leq |i| + |j| \leq r\},$$

where i and j are multi-indices. Let $f \in \mathrm{Diff}^r(M)$ ($r \geq 1$) have a heterodimensional cycle involving two saddles, one of which, denoted by O, has a homoclinic tangency. Consider any family $\{f_t \in \mathrm{Diff}^r(M)\}_{t \in V} \in C^{K,r}(V \times M, M)$, $0 \leq K < r \leq \infty$, such that $f_0 = f$ and the homoclinic tangency of O persists for all $t \in V$. By applying Berger's parablender construction to the arrayed blender system found near heterodimensional cycles, we show that there exist a ball $V' \subset V$ and an open set $\mathcal{U} \subset C^{K,r}(V \times M, M)$ arbitrarily close to $\{f_t\}$ such that $\mathcal{U}$ contains a dense subset where every family has, for every $t \in V'$, a locally persistent homoclinic tangency to the continuation of O. As an application, we show that families $\{f_t\}$, which have at some t_0 a simple homoclinic tangency to a weakly dissipative saddle, generically display infinitely many sinks for an open set of parameter values. Here the weak dissipation means that the saddle has no strong-unstable multipliers and contracts central volumes but is not sectionally dissipative. The theorem extends Berger's open set of families that generically display the Newhouse phenomenon with positive probability to all families that intersect the weakly-dissipative Newhouse domain.

High order homoclinic tangencies and universal dynamics for multidimensional diffeomorphisms

Dmitrii Mints *(Imperial college)*

Our research is aimed at studying the dynamics of smooth multidimensional diffeomorphisms from the Newhouse domain, that is, open regions in the space of maps where systems with homoclinic tangencies are dense. We prove that in the space of smooth and real-analytic multidimensional maps in any neighborhood of a map such that it has a bi-focus periodic orbit whose invariant manifolds are tangent, there exist open regions (which are subdomain of the Newhouse domain) where maps with high order homoclinic tangencies of corank 2 (invariant manifolds forming the tangency have a plane of common tangent vectors) are dense and maps having universal two-dimensional dynamics are residual. This is a joint work with D. Turaev.

Stable Motions in the Planar Circular Restricted 3-Body Problem

Otto Osterman *(University of Maryland)*

The N-body problem models the behavior of celestial objects undergoing mutual gravitational attraction. Some important problems in dynamical systems concern the long-term behavior of these systems. For example, the standard model of the solar system is to assume all planets have independent Kepler orbits around the Sun, ignoring their gravitational influence on each other, but in some cases, it is possible for the motions of smaller bodies to undergo large changes over time, or for chaotic motions to exist. My focus is on the planar circular restricted 3-body problem, modeling the motion of the Sun, a planet, and a massless asteroid moving in the same plane. For small mass ratios between the planet and the Sun, when the asteroid remains far from the planet, this system is approximated by independent Kepler orbits of the planet and the asteroid. However, if their orbits intersect, near-collisions between the planet and the asteroid are possible, which can lead to large changes in the orbit of the asteroid over a short time period. It has been shown that repeated near collisions can lead to chaotic motions (Bolotin, MacKay - 2000 ; Font, Nunes, Simó - 2002). My result, currently in preparation, is that there also exist stable motions in this configuration in which the asteroid repeatedly comes close to a collision with Jupiter, but not too close as to significantly divert its orbit.

Chaotic motions to L_4 in the restricted planar circular three-body problem

Donato Scarcella *(Universitat Politècnica de Catalunya)*

This talk deals with the Restricted Planar Circular 3-Body Problem (RPC3BP) close to the Lagrangian critical point L_4. The RPC3BP models the motion of a massless body under the gravitational influence of two massive bodies (the primaries having masses $\mu > 0$ and $1-\mu > 0$, respectively) performing circular orbits, assuming that the massless body moves in the same plane as the primaries. In a rotating framework that fixes the position of the primaries (a rotating framework with the same period of the primaries, also called synodic coordinates), the Hamiltonian of the RPC3BP is an autonomous two-degree of freedom Hamiltonian having five critical points L_1, L_2, L_3, L_4, and L_5 called the Lagrangian points. Before this symplectic reduction, these critical points correspond to periodic motions for the massless body with the same frequency as the primaries.

Let $\mu_1 = \frac{1}{2}\left(1 - \frac{1}{9}\sqrt{69}\right)$ be the Routh critical mass ratio. It is well known that if $0 < \mu < \mu_1$, the point L_4 is a center-type critical point, whereas for $\mu > \mu_1$ it is a complex saddle. This phenomenon is also called Hopf Bifurcation.

We are interested in values of the mass ratio $\mu > \mu_1$, where chaotic phenomena are expected to occur. To investigate this, we studied the two-dimensional stable and unstable manifolds associated with L_4, derived an asymptotic formula for the distance between these manifolds, and used it to prove the existence of Smale horseshoes close to L_4.

This study is a collaborative effort with Inmaculada Baldomá (UPC) and Pau Martín (UPC).

Statistics of prime orbits for expanding Thurston maps

Xianghui Shi *(Peking University)*

Expanding Thurston maps are topological models for certain non-uniformly hyperbolic rational maps without smoothness or holomorphicity assumptions. For these maps, we study a counting problem for primitive periodic orbits under a constraint on Birkhoff sums. Specifically, the constraint requires that for a given potential function, the Birkhoff sums along the periodic orbits lie in a prescribed family of shrinking intervals. For an eventually positive, Hölder continuous potential satisfying the strong non-integrability condition, we derive asymptotic estimates for the number of such constrained orbits. Our results resemble a local limit theorem, quantifying both the exponential growth rate of orbit counts and the fine-scale distribution governed by the potential function.

Tropical thermodynamic formalism

Yiqing Sun *(Peking University)*

We investigate the large deviation principle of equilibrium states by noticing that the logarithmic zero-temperature limit in LDP induces a tropical (max-plus) algebraic structure. We discuss this relation in Ruelle operators and Bousch operators and extend it this relation to their adjoint operators. It turns out that the uniqueness of eigen-objects of the Bousch operator governs the LDP whose rate function is given by an eigenfucntion and an eigen-density. Ideas and tools from tropical algebra and weak KAM theory give the representation of eigen-objects of the Bousch operator.

A temporal central limit theorem for irrational rotations

Hao Wu *(Universität Zürich)*

Dynamical systems are deterministic systems. However, in chaotic systems where the entropy is positive, the ergodic sums often behave similarly to the sums of independent random variables and satisfy the spatial central limit theorems (CLT) according. In zero-entropy systems, the spatial CLT often fails due to the lack of (fast) mixing properties. But in some cases, such as irrational rotations of bounded type, one can retrieve the central limit theorem if we study single orbit statistics, where we fix the starting point and randomise time, hence the word "temporal". In this talk, I will present an ongoing joint work with Bromberg and Ulcigrai, where we generalise their previous method of coding and Markov chains to obtain a temporal central limit theorem for a broader class of observables over bounded irrational rotations.